Dates for the Steps of *Star Axis*

Report for Charles Ross

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I. Introduction

This report focuses on the long stairway of Charles Ross's *Star Axis*, which is inclined at an angle equal to the latitude of its location in New Mexico and oriented exactly due north (Fig. 1 photos).[1] The 146 steps of the stairway ascend ~103 ft. As observers climb these steps, they face a 40-inch-diameter circular aperture at the very top, centered on the north celestial pole (NCP) and growing larger in angular size as it is approached. By accident the bright star Polaris is currently very close to the NCP and thus traces a small circle in the sky every day as the Earth rotates. Standing at the bottom of the stairway today, this circle closely coincides with the edge of the aperture.

Star Axis is aligned with the NCP, the extension of the rotation axis of the Earth. But the direction of that rotation axis (and the entire Earth, including *Star Axis*) slowly moves (precesses), causing the distance between the NCP and Polaris to continually change, and thus also the size of Polaris's daily circle. At present in 2017 CE the NCP is 0.66° from Polaris, and will be closest to Polaris (0.46°) in the year 2100 CE.[2] Over the next ~13,000 years it will move away to as much as ~50° and similarly, at ~13,000 years in the past, its maximum distance was about the same. Consequently, after 2100 CE as the centuries and millennia pass, the ex-polar star will steadily trace larger and larger daily circles around the NCP, which remains at the center

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[1] *Star Axis* is located at latitude 35.264°N and longitude 105.087°W.

[2] CE and BCE refer to Common Era and Before Common Era, replacing AD and BC. Dates in Tables are given according to the astronomical convention. Positive numbers are CE. Negative numbers, however, differ from BCE dates because of the lack of a year zero in our calendar. Thus +1 = 1 CE; 0 = 1 BCE; -1 = 2 BCE, etc.

of the aperture. But in order to see Polaris, future observers will need to climb the stairway, increasing the aperture's angular size so that it can encompass Polaris's larger circle (Fig. 2). Thus it is possible to associate a future (and a past) date with each step, and the entire structure of *Star Axis* becomes a long-term calendar intimately tied to the heavens.

The purpose of this report is to supply two accurate dates to be engraved on each step. In Section II we identify and explain nine effects that influence the changing position of Polaris on time scales varying from one year to ~40,000 years. Section III assesses the various sources of errors in dating the steps of *Star Axis*. Precession (including the change in the Earth's obliquity) and proper motion are found to be the only two phenomena of significance for the astronomical calculations. Section IV describes the stairway geometry in more detail, as well as how it affects our calculations. We end with the finally recommended values of dates for each step (Table 3).

II. Phenomena affecting the sky position of Polaris

There are nine principal phenomena that affect the apparent location of Polaris in the sky.[3] In order of size they are: the precession of Earth's axis combined with the changing obliquity of the ecliptic, proper motion, atmospheric refraction, aberration, nutation, Earth's polar motion, orbital motions of Polaris in its triple star system, and parallax. Each of these is now discussed in turn and summarized in Table 1. Figure 3 graphically compares the magnitude of the various effects.

Precession. Precession has by far has the greatest effect on Polaris's position in the sky, and is at the heart of the astronomical design of *Star Axis*. The Earth's rotational axis (and therefore the NCP) slowly wobbles because Earth is not a perfect sphere, but due to its rotation has an equatorial bulge (radius is 0.3% larger at the equator than at the poles). Like a spinning top, the axis is altered by external forces (Fig. 4). This motion is caused by gravitational forces between the Earth's equatorial bulge and (primarily) the Moon and Sun. The Earth's precessional period is ~26,000 years, describing roughly a circle on the sky of radius ~23° (Fig. 5). Figure 6 illustrates in greater detail the effect of precession on the size of Polaris's daily circle over the period 1700 to 2100 CE. The NCP will pass closest to Polaris on 20 March 2100, at a distance of 0.45° = 27′

[3] For the position of Polaris (J2000.0) we use right ascension = 37.95454°, declination = +89.26411°.

(Seidelmann 1992:502). Figure 7 shows the variation of the NCP-to-Polaris distance over the current ~26,000-yr quasi-cycle.

Changing obliquity of the ecliptic. This effect is usually considered part of "precession" and is incorporated into all standard precession equations; the next paragraph explains why we note it separately here. The obliquity of the ecliptic is the angle between the celestial equator, the projection of the Earth's equator on the sky, and the ecliptic, the plane of the Earth's orbit around the Sun. Changes in the obliquity are primarily caused by gravitational forces between the Earth's equatorial bulge and the planets. The current value is 23.44°, but it is now slowly decreasing at a rate of 47"= 0.0131° per century. Figure 8 shows its variation between 22.6° and 24.2° over the current precession cycle with a quasi-period of ~40,000 years.[4, 5]

The value of the obliquity determines the maximum and minimum declination of the sun as it cycles between summer and winter solstice (and is therefore important for seasonal effects). Our reason for distinguishing the slowly changing obliquity is its bearing bearing on other aspects of *Star Axis*, namely those parts of the structure based on an value for the obliquity assumed to be 23.50°.[6]

Proper motion. Each star is moving through the Galaxy with its own particular velocity with respect to the sun. Polaris thus steadily shifts in sky position, termed its proper motion, at a rate of 0.000767′ per year, as measured by the Hipparcos satellite.[10] In 13,000 years, Polaris's sky position will have changed by ~10′ because of this motion. For any given epoch, the effect may increase or decrease the distance of Polaris from the NCP, called the North Polar Distance

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[4] This changing value of the obliquity is why the precession motion on the sky is not exact in its circularity, size or periodicity. The described path is actually quasi-circular, a slight spiral with a slowly changing radius that oscillates back and forth with a quasi-period. This also means that the precessional shift of a star's position is *not* symmetric going forwards versus going backwards in time (as can be seen at the extremes of Fig. 7).

[5] Official astronomical terminology has recently changed (Urban and Seidelmann 2013). "Precession" in this report is properly called "general precession", consisting of two components: (1) that due primarily to the Moon and Sun (formerly called "lunisolar precession", now "precession of the equator"), and (2) the much smaller component due to the planets (formerly called "planetary precession", now "precession of the ecliptic").

[6] The changing obliquity potentially supplies an elegant way for any future archaeologists to neatly date the construction of *Star Axis.* All that is required is an accurately scribed 23.44° angle on a wall somewhere. For example, an accuracy of 0.01° would date the structure to about one century. If the two sides of the angle can be ~50 ft in length, the desired 0.01° accuracy for the angle leads to a required surveyed accuracy of 0.01 inch in the relative coordinates of the three points defining the angle.

[10] At the distance of Polaris (430 light-yr), this proper motion amounts to a speed of 29 km/sec.

(NPD). Figure 9 shows how the value of NPD is affected by proper motion.The discussion in Sec. III indicates that proper motion must also be included in our calculations if the steps are to be dated to better than one year accuracy.

Table 1. Phenomena affecting the sky position of Polaris

Phenomenon	**Period (years)**	**Amplitude (arcsec)**	**Comments**
Precession	~26,000	~85,000″ (~23.5°)	"Wobble" of Earth's pole due to gravity of Moon & Sun acting on Earth's bulge - central to *Star Axis* concept
(Obliquity Change)	~40,000	~7200″ (~2°)	(Included in precession equations) "Tilt" of Earth's pole varies due to gravity of planets acting on Earth's bulge
Proper Motion	—	—	Not periodic, accumulates linearly; space motion of star amounts to10′ over 13,000 years
Refraction	—	—	Not periodic, constant offset; 1.4′ bending of light in Earth's atmosphere
Aberration	1	20″	Apparent shift in star position due to Earth's motion
Nutation	19	17″	"Nodding" due to Moon's tilted orbit
Earth's Polar Motion	~1	0.3″	Earth's rotation axis wanders quasi-periodically due to internal mass redistributions
Orbital Motions	30	~0.03″	Polaris's motions as part of a triple-star system
Parallax	1	0.007″	Apparent shift in star position due to Earth changing in position as it orbits the Sun

Refraction. The Earth's atmosphere bends the light from Polaris such that its apparent location is always farther above the horizon than the actual location (Fig. 10). At *Star Axis*'s latitude of 35.26° Polaris is at an altitude of ~35° and the typical amount of refraction is ~1.4′ (somewhat variable with temperature and humidity), meaning that Polaris appears 1.4′ higher in the sky than it actually is. Further, the entire circle traced by Polaris appears slightly higher in the sky (thus not exactly centered in the aperture) than it would be with no atmosphere. This fixed offset will be more noticeable when the aperture appears smallest, i.e., from the first few dozen steps.[11]

Aberration of starlight. Aberration arises as a result of the finite speed of light and the variable velocity vector of the Earth in its annual orbit. An analogy is helpful. When standing in the rain

[11] For distant dates in the future and past, when Polaris's circle becomes much larger, Polaris will encounter variable and much larger refraction effects in different portions of its daily circle, especially as it gets closer to the horizon. Today's almost perfect "circle" will thus become slightly "squashed" in its lowest portion.

(falling straight down), you hold an umbrella directly above to intercept the drops, but if you are walking fast, you must tilt the umbrella in the direction of your motion to avoid getting wet. Likewise astronomers must tilt their telescopes because the apparent direction of the starlight (raindrops) changes depending on the momentary direction and size of Earth's orbital velocity. The effect can be as large as ±20″, and for Polaris, causes it to describe a small ellipse on the sky every year (Fig. 11).

Nutation. As the direction of Earth's rotation axis (and the NCP) travels on the precession circle, it "nods" up and down by ±9″ and "backwards and forwards" by ±17″every 18.6 years because the plane of the Moon's orbit (and gravitational force) is tilted with respect to the Earth's equator and changes its orientation ("line of nodes") with an 18.6 year period.

Earth's polar motion. The rotation axis of the Earth (and hence the NCP) changes very slightly relative to the body of the Earth. This is due to adjustments in the near-surface interior mass, largely caused by seasonal effects in the atmosphere. Typical amplitudes are ~0.3″ with quasi-periods of about one year.

Orbital motions of Polaris in its triple-star system. Polaris has two companion stars and a resultant orbital motion and periodic oscillation in sky position due to their gravitational influence (Fig. 12). The star generally called "Polaris" is ~600 times brighter than its companions and is properly termed "Polaris A." Polaris A and its near companion, Polaris Ab, orbit about a common center of mass, causing A's position to shift as much as ±0.03″ every 29.6 years (Evans et al. 2008). Moreover, A and Ab are in a binary system with Polaris B, a more distant third star that is also likely gravitationally bound in the system. (A+Ab) must have a very long orbital period with B, not yet able to be determined.[12]

Parallax. As the Earth moves from one side of its orbit to the other, any star's position appears to shift in the opposite sense due to our changing perspective - the nearer the star, the greater the

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[12] Polaris A ("Polaris") is a 4.5 $M_\odot$ supergiant star (where $M_\odot$ is 1.0 solar mass) of apparent magnitude $m = 2.0$. It is also a Cepheid variable star, varying by as much as ±0.1 mag every ~4.0 days. Polaris Ab is a 1.3 $M_\odot$ star of $m = 9.2$ and was discovered in 1929. Polaris B, a 1.4 $M_\odot$ star of $m = 8.7$, was discovered by William Herschel in August 1779.

apparent annual shift (Fig. 13). By measuring the tiny annual shift and knowing the Earth-Sun distance, we can deduce extremely accurate distances to a star. The annual shift for Polaris is ±0.00756″ (Hipparcos satellite), putting it at a distance of 430 light-years.

III. Uncertainties in the step dates

How well must we locate Polaris in future (and past) millennia in order to accurately date each step? Table 1 and Fig. 3 summarize the effects discussed above. To judge whether or not any effect is significant for determining step dates to the nearest year and thus must be included in the calculations, we need to know how much a given change in Polaris's position affects the basic design goal of *Star Axis*, namely that when standing on the stair corresponding to the year of observation, a person should observe the nightly circle of Polaris coincide with the perimeter of the aperture at the top of the stairway.

The physical radius of the aperture is 20 inch, but Ross has wisely decided to have calculations done for a "virtual" aperture of radius 18.875 inch, creating a perimeter annulus of width 1.125 inch (hereafter taken as 1.12 inch). A certain amount of error can thus be accomodated, and the observer will still be able to see the entirety of Polaris's daily circle close to the periphery of the aperture. Table 2 shows the angular size of the 1.12 inch annulus as seen from the steps; any errors must be smaller than this.

The basic formula for the angular radius ρ of the aperture as seen by an observer's eyeball from a Step is (Fig. 14):

$$\tan \rho = 18.875 \text{ inch} / d \ ,$$

where d is the distance from the eyeball to the aperture center.

In general, errors in the calculated radius or location of Polaris's daily circle, and thus the dates on a step, can potentially arise from (a) incorrect positioning of the observer's eyes while standing on a step (essentially an error in the *location* of the vertex of the angle ρ, (b) incorrect dimensions d for the stairway structure, or (c) incorrect astronomical calculations for NPD at a given date. Each of these is now considered.

(a) Position of the observer's eyes. Although not a problem in dating the steps, it is paramount for the observer to center him/herself on the center of a step and place his/her eyeball at Ross's design height of 66.0 inch above the tread. An error in eyeball placement of 1.12 inch or more in any direction will cause the aperture to shift (in the opposite sense) by 1.12 inch or more; this statement is correct no matter which step the observer is on. For an observer whose eye is offset by > 1.12 inch, a portion of the daily circle of Polaris will pass outside the aperture, and, depending on the season and time of night, Polaris will sometimes not be visible.

It would seem to be difficult for a person to situate his/her eyes accurately enough - *within about one inch* - without an aid. This could take the form of a pole of the correct height, perhaps having a small ring on the top (tilted at the 35° upwards angle to the aperture). To guarantee that the pole is vertical, three feet on the bottom would suffice. In addition, a central mark on the tread of each step will be essential, showing where the pole should be placed; this mark would be very useful even if no pole is used.

(b) Stairway structure dimensions. All adopted structural dimensions have been supplied to us by Ross (Fig. 15). Although we work with formal dimensions to the nearest 0.01 inch, the structure is certainly not as accurate as this in its largest dimensions. An error ε in the assumed distance d to the aperture from any given step corresponds to an error in tan ρ of $18.875\varepsilon / d^2$. For an assumed error ε of ±0.1 inch, it turns out that the resultant errors in ρ and thus step dates are small for all but the last dozen steps, as shown in Table 2. Errors in step dates increase from ~0.5-1.0 yr for Steps #135-140 to tens of years for Steps 145-147. Larger values of ε will cause proportionately larger errors in dates.

Table 2. Estimated errors of step dates

Step Number	Date > 2100	Date < 2100	Distance to aperture d (inch)	Aperture angular radius ρ (deg)	1.12 inch annulus: Tolerable error (arcmin)	Error of 0.1 inch in d: Error in ρ (arcmin)	Error of 0.1 inch in d: Error in step date (years)	Est. error in NPD: Error in calculated NPD (arcmin)	Est. error in NPD: Error in step date (years)
1[14]	2137	2067	2162.86	0.5000	1.8	< 0.1	< 0.1	< 0.02	< 0.1
35	2184	2020	1659.92	0.6515	2.3	< 0.1	< 0.1	< 0.02	< 0.1
70	2248	1955	1148.04	0.9419	3.4	< 0.1	< 0.1	< 0.02	< 0.1
105	2393	1811	636.17	1.6994	6.1	< 0.1	< 0.1	< 0.02	< 0.1
140	3648	565	124.30	8.6345	31	0.5	~1[15]	< 0.1	< 0.3
141	3854	361	109.67	9.7651	35	0.6	~2	< 0.1	< 0.3
142	4123	96	95.04	11.2319	41	0.8	~3	< 0.2	< 0.6
143	4489	-263	80.42	13.2080	48	1.2	~4	< 0.5	~1
144	5016	-778	65.80	16.0062	59	1.7	~6	~1	~3
145	5840	-1580	51.17	20.2462	75	3	~10	~2	~7
146	7310	-2980	36.54	27.3136	110	7	~30	~10	~50
147[16]	14400	-10900	15.84	50.0000	240	11	~50	~200	~1000?

Notes:

Col. 6 is the largest tolerable error (from any of the 3 causes in Sec. III) given that the aperture radius has been defined as 18.875 inch when it is actually 20.00 inch.

Col. 7 is the error in aperture angular radius ρ caused by an error of 0.1 inch in the distance d from a Step to the aperture.

Col. 8 is the error in Step date caused by an example 0.1 inch error in Col. 7.
Col. 9 is the estimated error in the calculated NPD values from the precession and proper motion calculations.
Col. 10 is the error in Step date caused by the error in Col. 9.

[14] "Step" (or Position) #1 is *defined* as having $\rho = 0.5000°$.
[15] The error in step date becomes > 0.5 yr at Step #135.
[16] "Step" (or Position) #147 is *defined* as having $\rho = 50.0000°$.

(c) Calculated Polaris positions. From the bottom of the stairway an angle of 1.8′ corresponds to 1.12 inch at the distance of the aperture (2162.86 inch; see Fig.15 and Table 2). For the first three-quarters of the stairway (up to Step #105, corresponding to dates within ~300 years from the present), errors in the NPD of Polaris as large as 1.8′ are acceptable; this is easily accomplished with our precession equations. As one ascends the stairway into a domain of more remote dates, likely errors in the precession equations slowly increase, and become very large for the final few steps (Table 2). But for those steps any given angle also corresponds to fewer inches at the aperture and therefore larger errors can fortunately be accomodated by the annulus. Table 2 lists sky position errors equivalent to 1.12 inch at the aperture - these are the largest tolerable in order to see Polaris. In fact the estimated errors in our calculated positions are much less, with only the exception of Step #147, where the aperture subtends an angle of 50.00° and the corresponding dates are 12-13 millennia from now. For such distant dates positions calculated from models of precession developed by various experts diverge by by as much as 3° (Vondak et al. 2011). In the far future we simply do not know accurately where Polaris (or any other star) will appear as seen from the Earth on a given date; or better stated, we do not know within ~3° where the Earth's rotational axis will be pointing on such distant dates.[17, 18]

We have used the precession expressions found in Sec. 2 of Simon et al. (1994).[19] The estimated errors in our positions (Table 2) using these equations are taken from Vondak et al. (2011:Fig. 13), where a very useful comparison is made of positions calculated with several state-of-the-art precession models for dates as far as ±15000 years from the present. We then use these to estimate the resulting error in dating the steps. It can be seen that our calculated step dates are accurate to better than one year for dates nearer than two millennia to the present. For the last three steps, however, dates become very remote and precession theory becomes much less precise. For this reason all steps through #144 are given to the nearest year, but #145 and #146 only to the nearest decade, and #147 to the nearest century.

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[17] The major uncertainties are in the slowly changing shape of the Earth and in the dissipation of energy by tidal friction near the Earth's surface.

[18] Note that when the NCP is > 35.3° from Polaris, Polaris will cease to be a circumpolar star at the *Star Axis* site. This happens at times after 9200 and before -4800. Only Step #147 is relevant for such distant epochs, because only from this last step can an observer see the ground through the aperture.

[19] For an accessible, but still technical, treatment of precession, Chap. 20 of Meeus (1991) is recommended.

Proper motion for Polaris has been extrapolated into remote times using standard proper motion geometry. The effect of proper motion on the calculated NPD values (see Fig. 9) becomes significant for dating the steps from approximately Step #70 upwards. Unlike precession, proper motion can be very reliably extrapolated out to ±13,000 yr.

In summary, regarding accuracy:

— The exact position of the observer's eyes is a critical factor needing attention to aid the observing experience, but could be greatly improved with a portable positioning device.

— Uncertainties in the structure's dimensions may be the limiting factor in dating the steps. If the stairway is accurate to ±0.1 inch, its error contribution to the dates is > 0.5 yr for Step #130 and above (more than ±600 yr from the present).

— For Polaris's sky position only precession and proper motion have needed to be calculated. We derive an accuracy of < 1 yr for the step dates for ±2000 yr from the present, increasing to ~10 yr for ±4000 yr (up to Step #145).

— The accuracy of the precession and proper motion equations are sufficient to guarantee that Polaris's circle, as seen by an observer from the calculated step, will always be located near the periphery of the aperture (except Step #147).

IV. Last steps for the dating

"Step #1" of *Star Axis* is not actually a step, but a designated spot on the ground that is 20.31 inch (diagonally) from Step #2, which is defined as the middle of the tread of the first physical step.[20] By *diagonally* we mean as measured along the angle of the stairway (35.267°, the site latitude) up to the center of the aperture. As discussed above, all distances are referenced to an assumed person's eyeball height of 66.0 inches when standing on a step. The diagonal step-to-step distances from Step #2 to Step #146 are each 14.625 inch. Finally, the diagonal distance

[20] Two other practical dimensions are shown in Fig. 15, namely the distance of the actual position of "Step" #1 from the riser up to Step #2 (10.61 inch), and the distance of the actual position of "Step" #147 from the riser down to Step #146 (10.94 inch).

from Step #146 to Step #147 (which once again, is actually a spot on a platform) is 20.71 inch, and the distance from the viewer's eyeball on Step #147 to the center of the aperture is only 15.84 inch.

The distance from each Step # N to the aperture is (Fig.15):

$N = 1$	$d = 20.31 + 144 \times 14.625 + 36.55 = 2162.86$ inch
$N = 2$ through 146	$d = (146 - N) \times 14.625 + 36.55$
$N = 147$	$d = 15.84$ inch

The position of Step #1 has been chosen such that the radius ρ of the aperture subtends an angle of exactly 0.5000°. This NPD value occurs roughly symmetrically about the year 2100 (when Polaris's NPD is at a minimum (Fig. 6)), namely for the years 2137 and 2067 CE (Table 2). Thus *Star Axis* is actually centered on the date 2100 CE and "future" and "past" are better understood with respect to that date. Figure 16 shows the overall scheme of how the dates run on the stairway. On "one side" of the steps, dates run forward as one ascends from 2137 (Step #1) to 14,400 (Step #147), while on the other side they run backward from 2067 to -10,900. The year of this report, 2017, is on Step #37. There are no steps corresponding to the years 2068 through 2137 CE, when the NPD of Polaris is less than 0.50°; the smallest is 0.45° (Fig. 6). During those years, as seen from Step #1, Polaris will describe a somewhat smaller daily circle (by 0-10%) than the assumed aperture size.

The position of Step #147 has been chosen such that ρ encompasses the maximum NPD values encountered over the current precession cycle. ρ is taken to be 50.00°, which is a "safe" value a bit larger than the actual extreme (which is known only to a low accuracy of ~45° to 50°).The reported dates for Step #147 have been calculated for the maximum nominal values of NPD.

The calculations were carried out as follows. The precession and proper motion equations were solved to calculate the NPD of Polaris at one-year intervals. From this list we found, for each step, which pair of years (interpolated to ~0.1 year) had NPD values that best matched ρ, the calculated angular radius of the aperture.

Finally, dates are to be engraved on the steps only in whole years. This has two consequences. First, a rule must be adopted as to how to round off calculated dates. The adopted rule is that all step dates are rounded to the nearest **mid-point** of a year (for example, 2017.50 in astronomical notation), not to the nearest start of a year (2017.00 or 2018.00). The logic is that future visitors to Star Axis will be visiting during the middle portion of calendar years, not during the winter! Thus calculated dates of 2017.6 through 2017.9 are rounded to 2017, not to 2018. The second consequence is that the step-to-step increments seem unusual, especially noticeable for the first 100 steps, because the average step-to-step date increments turn out to be in the range of 1 to 2 years. For example, as Table 3 shows, the step-to-step increments going backward in time from Step #1 are (in years) 2, 2, 1, 2, 1, 1, 2, 1, etc. Figure 17 nicely illustrates this round-off effect.

Finally, at last, Table 3 below lists the two dates to be engraved on each of the 147 steps.

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Table 3. Dates for the 147 Steps of *Star Axis*

(Sullivan & Thorp, 2017)

Step Number	Aperture distance *d* (inch)	Aperture angular radius *ρ* (deg)	Step No.	Date > 2100	Date < 2100
1	2162.86	0.5000	1	2137	2067
2	2142.54	0.5047	2	2138	2065
3	2127.92	0.5082	3	2140	2063
4	2113.29	0.5117	4	2141	2062
5	2098.67	0.5153	5	2143	2060
6	2084.04	0.5189	6	2144	2059
7	2069.42	0.5226	7	2146	2058
8	2054.79	0.5263	8	2147	2056
9	2040.17	0.5301	9	2148	2055
10	2025.55	0.5339	10	2150	2054
11	2010.92	0.5378	11	2151	2052
12	1996.30	0.5417	12	2152	2051
13	1981.67	0.5457	13	2154	2050
14	1967.05	0.5498	14	2155	2048
15	1952.42	0.5539	15	2156	2047
16	1937.80	0.5581	16	2158	2046
17	1923.17	0.5623	17	2159	2044
18	1908.55	0.5666	18	2160	2043
19	1893.92	0.5710	19	2162	2042
20	1879.30	0.5754	20	2163	2040
21	1864.67	0.5800	21	2164	2039
22	1850.05	0.5845	22	2166	2038
23	1835.42	0.5892	23	2167	2036
24	1820.80	0.5939	24	2168	2035
25	1806.17	0.5987	25	2170	2034
26	1791.55	0.6036	26	2171	2032
27	1776.92	0.6086	27	2172	2031
28	1762.30	0.6136	28	2174	2029
29	1747.67	0.6188	29	2175	2028
30	1733.05	0.6240	30	2177	2027
31	1718.42	0.6293	31	2178	2025
32	1703.80	0.6347	32	2179	2024
33	1689.17	0.6402	33	2181	2022
34	1674.55	0.6458	34	2182	2021
35	1659.92	0.6515	35	2184	2020
36	1645.30	0.6573	36	2185	2018

37	1630.67	0.6632	37	2186	2017
38	1616.05	0.6692	38	2188	2015
39	1601.42	0.6753	39	2189	2014
Step Number	Aperture distance *d* (inch)	Aperture angular radius ρ (deg)	Step No.	Date > 2100	Date < 2100
40	1586.80	0.6815	40	2191	2012
41	1572.17	0.6878	41	2193	2011
42	1557.55	0.6943	42	2194	2009
43	1542.92	0.7009	43	2196	2008
44	1528.30	0.7076	44	2197	2006
45	1513.67	0.7144	45	2199	2004
46	1499.05	0.7214	46	2200	2003
47	1484.42	0.7285	47	2202	2001
48	1469.80	0.7357	48	2204	2000
49	1455.17	0.7431	49	2205	1998
50	1440.55	0.7507	50	2207	1996
51	1425.92	0.7584	51	2209	1994
52	1411.30	0.7662	52	2211	1993
53	1396.67	0.7743	53	2213	1991
54	1382.05	0.7825	54	2214	1989
55	1367.42	0.7908	55	2216	1987
56	1352.80	0.7994	56	2218	1985
57	1338.17	0.8081	57	2220	1983
58	1323.55	0.8170	58	2222	1982
59	1308.92	0.8262	59	2224	1980
60	1294.30	0.8355	60	2226	1978
61	1279.67	0.8450	61	2228	1976
62	1265.05	0.8548	62	2230	1974
63	1250.42	0.8648	63	2232	1971
64	1235.80	0.8750	64	2234	1969
65	1221.17	0.8855	65	2236	1967
66	1206.55	0.8963	66	2238	1965
67	1191.92	0.9072	67	2241	1963
68	1177.30	0.9185	68	2243	1960
69	1162.67	0.9301	69	2245	1958
70	1148.05	0.9419	70	2248	1955
71	1133.42	0.9541	71	2250	1953
72	1118.80	0.9665	72	2253	1950
73	1104.17	0.9793	73	2255	1948
74	1089.55	0.9925	74	2258	1945
75	1074.92	1.0060	75	2261	1943
76	1060.30	1.0199	76	2264	1940
77	1045.67	1.0341	77	2266	1937
78	1031.05	1.0488	78	2269	1934
79	1016.42	1.0639	79	2272	1931

80	1001.79	1.0794	80	2275	1928
81	987.17	1.0954	81	2278	1925
Step Number	Aperture distance *d* (inch)	Aperture angular radius ρ (deg)	Step No.	Date > 2100	Date < 2100
82	972.54	1.1118	82	2282	1922
83	957.92	1.1288	83	2285	1918
84	943.29	1.1463	84	2288	1915
85	928.67	1.1644	85	2292	1911
86	914.04	1.1830	86	2296	1908
87	899.42	1.2022	87	2299	1904
88	884.79	1.2221	88	2303	1900
89	870.17	1.2426	89	2307	1896
90	855.54	1.2639	90	2311	1892
91	840.92	1.2858	91	2315	1888
92	826.29	1.3086	92	2320	1884
93	811.67	1.3321	93	2324	1879
94	797.04	1.3566	94	2329	1875
95	782.42	1.3819	95	2333	1870
96	767.79	1.4082	96	2338	1865
97	753.17	1.4356	97	2344	1860
98	738.54	1.4640	98	2349	1855
99	723.92	1.4936	99	2355	1849
100	709.29	1.5243	100	2360	1843
101	694.67	1.5564	101	2366	1837
102	680.04	1.5899	102	2372	1831
103	665.42	1.6248	103	2379	1825
104	650.79	1.6613	104	2386	1818
105	636.17	1.6995	105	2393	1811
106	621.54	1.7394	106	2400	1803
107	606.92	1.7813	107	2408	1796
108	592.29	1.8253	108	2416	1788
109	577.67	1.8714	109	2425	1779
110	563.04	1.9200	110	2433	1770
111	548.42	1.9712	111	2443	1761
112	533.79	2.0251	112	2453	1751
113	519.17	2.0821	113	2463	1741
114	504.55	2.1424	114	2474	1730
115	489.92	2.2063	115	2486	1718
116	475.30	2.2741	116	2499	1706
117	460.67	2.3463	117	2511	1693
118	446.05	2.4231	118	2525	1679
119	431.42	2.5051	119	2540	1664
120	416.80	2.5929	120	2556	1648
121	402.17	2.6871	121	2573	1631
122	387.55	2.7883	122	2591	1613

123	372.92	2.8975	123	2611	1593
Step Number	Aperture distance *d* (inch)	Aperture angular radius ρ (deg)	Step No.	Date > 2100	Date < 2100
124	358.30	3.0156	124	2633	1572
125	343.67	3.1436	125	2656	1549
126	329.05	3.2831	126	2681	1524
127	314.42	3.4354	127	2708	1496
128	299.80	3.6026	128	2739	1466
129	285.17	3.7868	129	2772	1433
130	270.55	3.9909	130	2808	1397
131	255.92	4.2181	131	2849	1356
132	241.29	4.4728	132	2895	1310
133	226.67	4.7601	133	2947	1259
134	212.04	5.0867	134	3006	1202
135	197.42	5.4614	135	3073	1133
136	182.79	5.8953	136	3152	1056
137	168.17	6.4039	137	3243	965
138	153.54	7.0081	138	3353	856
139	138.92	7.7374	139	3485	726
140	124.29	8.6348	140	3648	565
141	109.67	9.7653	141	3854	361
142	95.04	11.2322	142	4123	96
143	80.42	13.2085	143	4489	-263
144	65.79	16.0069	144	5016	-778
145	51.17	20.2474	145	5840	-1580
146	36.54	27.3157	146	7310	-2980
147	15.84	50.0000	147	14400	-10900

Notes

- All year values are rounded off to the nearest mid-year point, i.e., a calculated date that falls anywhere within a calendar year is listed as that year's value.

- Negative dates use the astronomical convention. For example -263 = 264 BCE.

- Step #1 is a position in front of the first step, defined such that the aperture has an angular diameter of 0.50000°.

- Step #147 is a position defined such that the aperture has an angular diameter of 50.0°. But because the NPD of Polaris never reaches as much as 50.0°, the listed dates are those when the NPD is calculated to be maximum.

Figures

Figs. 1. Photos of Star Axis (Credit: Charles Ross). (a) Overview of the structure (looking northeast), with part of the stairway visible. (b) From the bottom of the stairway, looking upwards toward the aperture, visible as a blue circle ~180 ft away. (c) Close-up view from within the Star Chamber of the final steps and aperture (diameter 40 inches). (d) A time exposure of star trails, showing the alignment of Star Axis with the North Celestial Pole (NCP), as well as the fact that Polaris (the bright, very short trail at ~"4 o'clock") is not located exactly at the NCP. This photo was taken at an early stage of construction.

Fig. l(b).

Fig. l(c).

Fig. l(d).

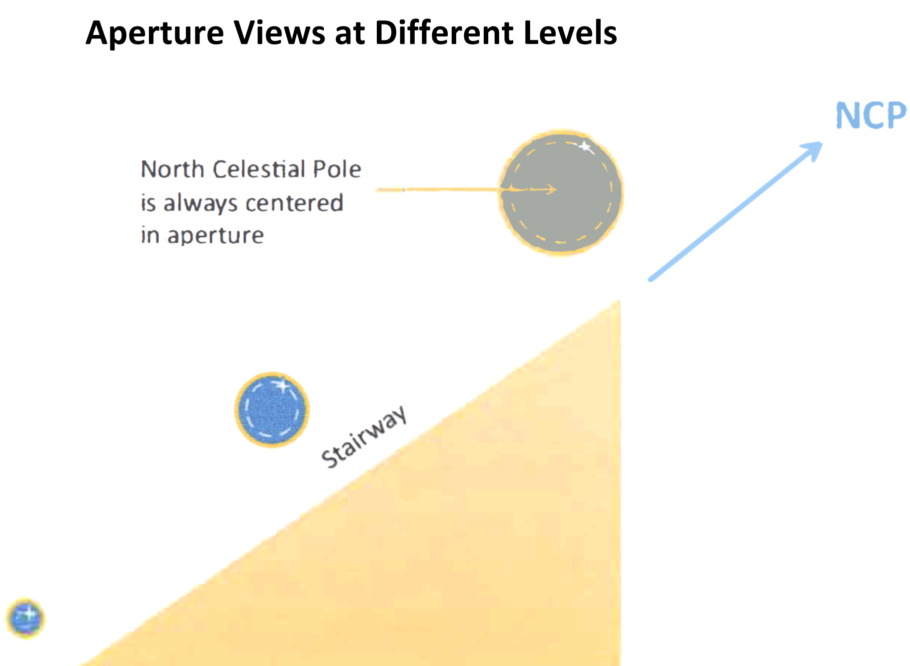

Fig. 2. The daily path of Polaris as seen in the aperture from different points on the stairway. As the viewer climbs the stairs and the distance to the aperture decreases, the angular size of the aperture increases. Lower steps correspond to the present epoch, when Polaris is close to the NCP (which is at the center of the aperture). Higher steps correspond to dates more in the past or future when Polaris is much farther from the NCP.

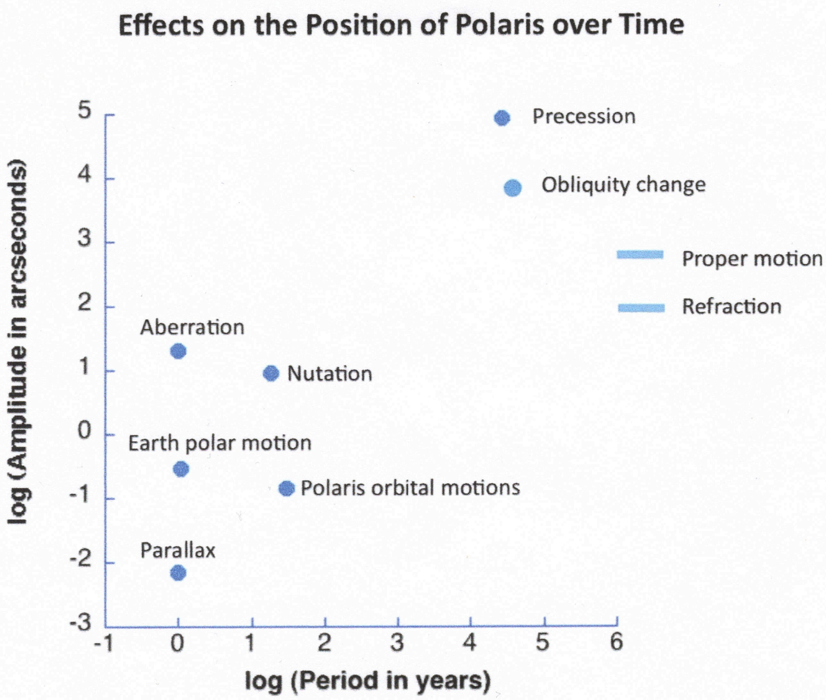

Fig. 3. The magnitude of the various phenomena affecting the apparent position of Polaris versus the approximate period of each effect. Proper motion and refraction are not periodic. Only general precession (which includes the changing obliquity of the ecliptic) and proper motion are significant for dating the stairs of *Star Axis*.

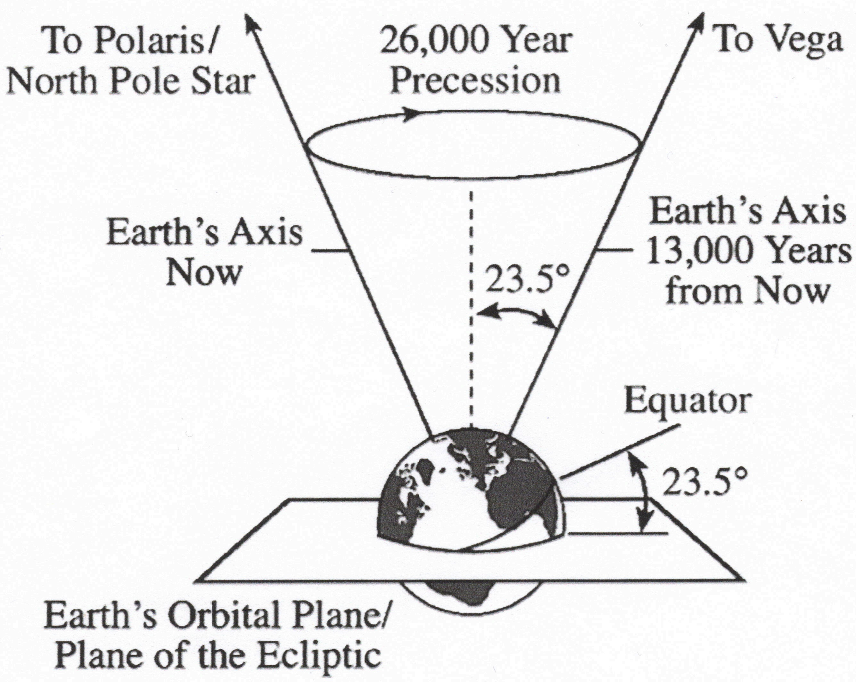

Fig. 4. Precession of Earth's axis and the NCP, as seen from space. Polaris is roughly in the current direction of the extension of Earth's axis; Vega will assume this role in ~12,000 years. The center of the precession cone is the North Ecliptic Pole (not labelled), which is perpendicular to the ecliptic plane (the plane of Earth's orbit). Credit: erenow.com/ancient/ancient-astronomy/18.html

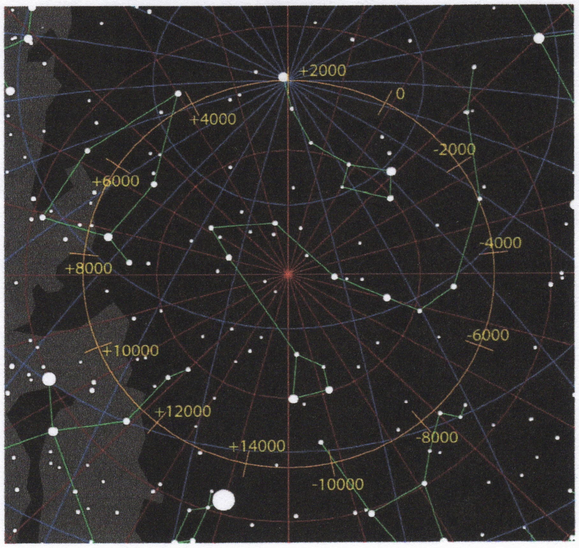

Fig. 5. The precession path of the NCP projected on the sky. The quasi-circle is centered on the North Ecliptic Pole, has a radius of ~23°, and a period of ~26,000 years. The dates extend from the year -10,000 to the year +14,000. At the current time the closest bright star to the NCP is Polaris; in the year +14,000 it will be much brighter Vega. Credit: Wikipedia "Axial precession"

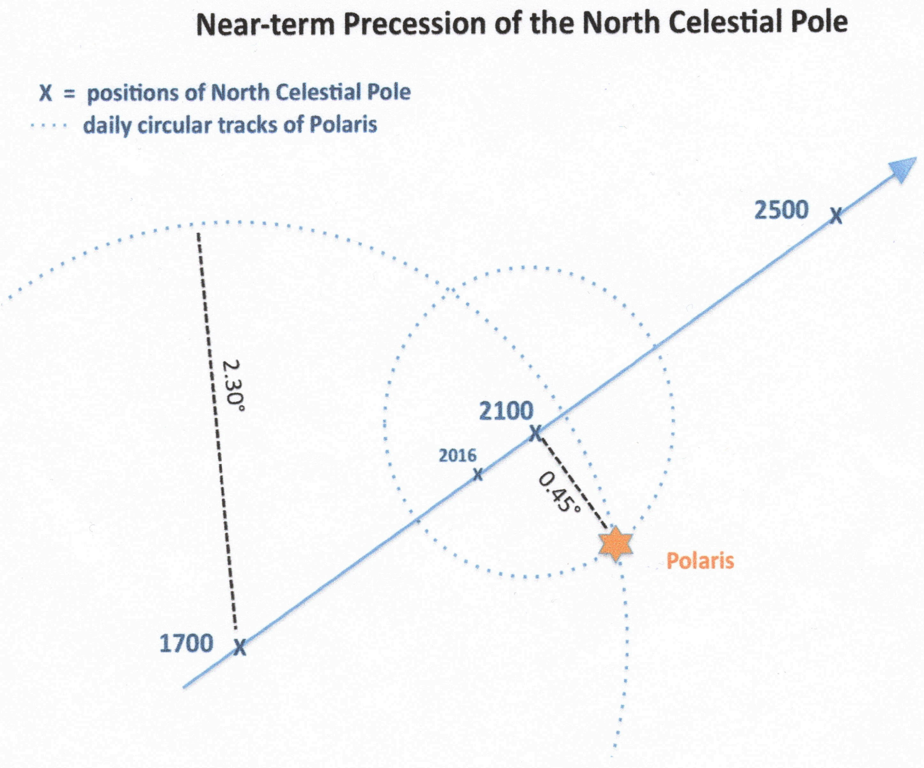

Fig. 6. The effect of precession on the daily circular track of Polaris from 1700 to 2500 CE. The straight blue arrow shows the movement of the NCP as it passes by Polaris, which circles daily about the NCP's ever-drifting position. Daily circles (dotted lines) are shown for the years 1700 and 2100, which is the closest that the NCP approaches Polaris.

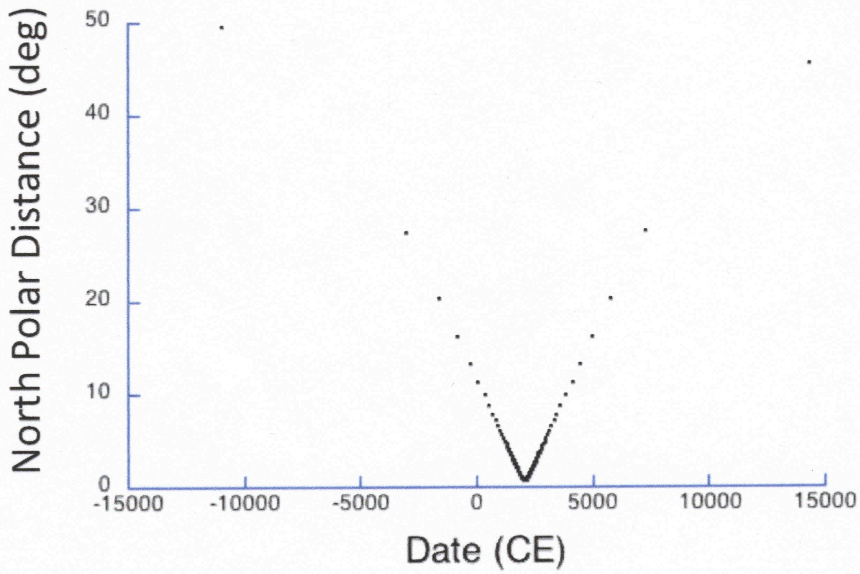

Fig. 7. The calculated North Polar Distance of Polaris over the current precession cycle. In order to find the dates for each step, these values are matched with the angular size of the *Star Axis* aperture radius as seen from each step. The minimum occurs on 20 Mar 2100 CE.

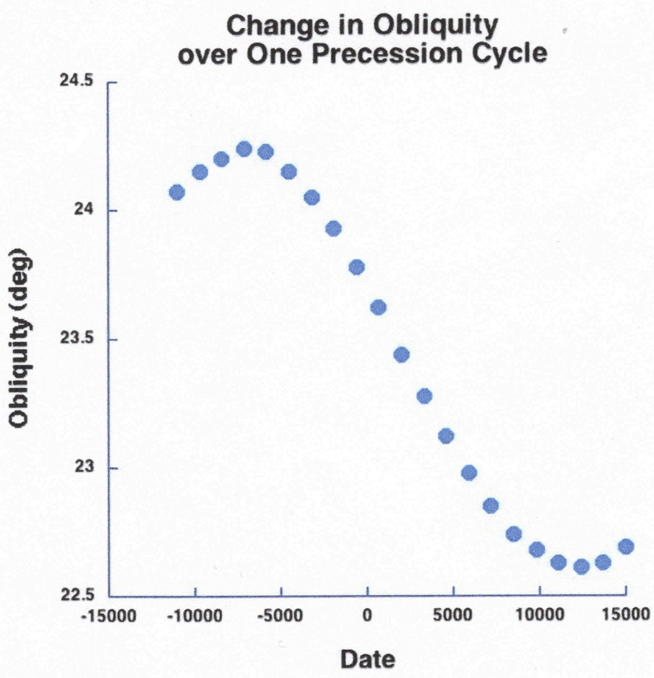

Fig. 8. The changing obliquity of the ecliptic over the current precession cycle. The current value is 23.44° , decreasing at a rate of 0.0131° per century. The overall quasi-period is ~40,000 years with an amplitude of ~2.0° , varying roughly between 22° and 24°. Credit: Plot derived from Chart 4 found at mb-soft.com/public3/equatime.html, which is in turn based on the work of Laskar (1999).

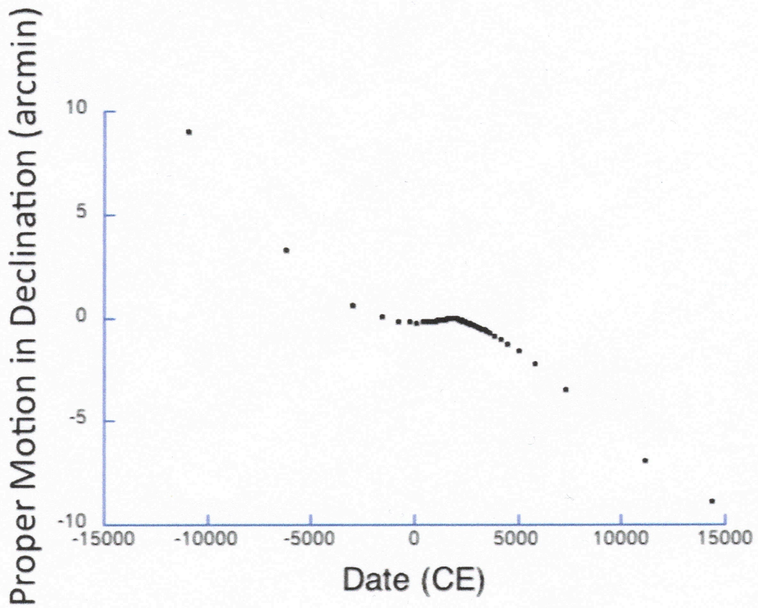

Fig. 9. The north-south component of proper motion (with respect to the year 2000 CE) for each of the 147 steps. This component affects the value of the North Polar Distance (NPD) and thus the proper dates for the steps.

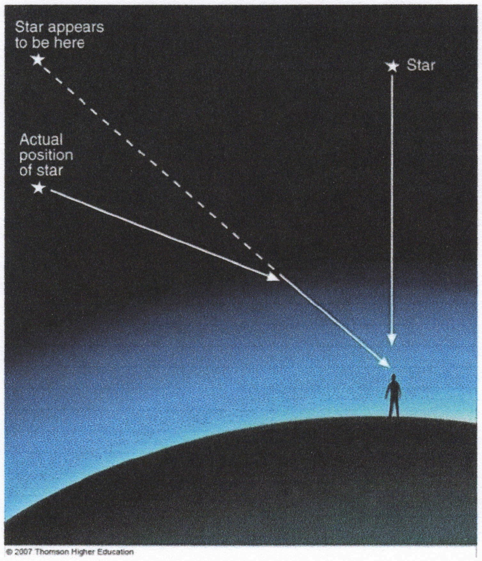

Fig. 10. The effect of refraction. When light from a star enters the atmosphere, the gaseous medium bends the light "downwards" so that it enters an observer's eye at a higher angle above the horizon, creating an apparently higher altitude for the star. Credit: Wikipedia "Atmospheric refraction".

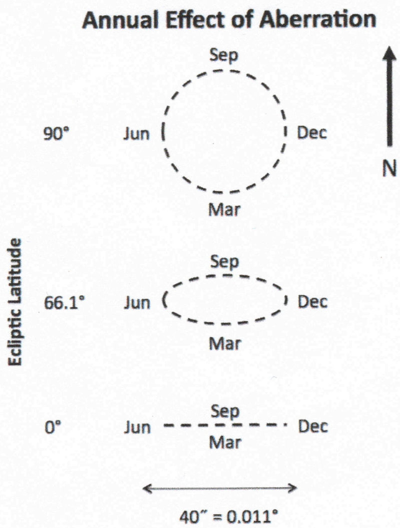

Fig. 11. The effect of aberration of light on a star's apparent position over a year; ecliptic latitudes for the three cases are listed on the left (66.1° corresponds to Polaris). The assumed ecliptic longitude is 270°. The different shapes correspond to the annually oscillating transverse component of the Earth's orbital velocity as seen from the star. Credit: Wikipedia "Aberration of light"

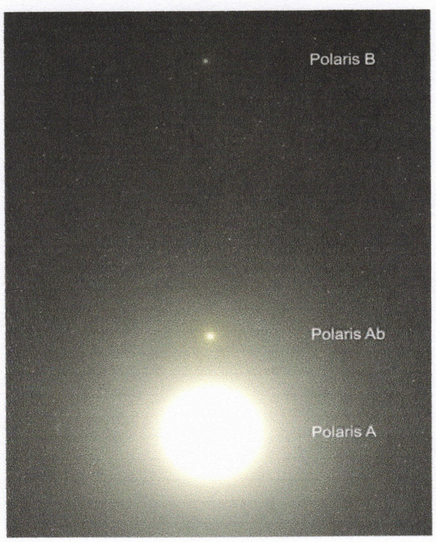

Fig. 12. Artist's conception of Polaris's triple-star system; the apparent image size of each star is proportional to its brightness. The separation of Polaris Ab from A was 0.17″ in 2006. Due to gravitational interactions, the smaller companion star Polaris Ab causes Polaris A to move in an ellipse with a periodicity of 29.6 years and a size of 0.06″. Polaris B is also part of the system and causes another small motion of Polaris A of very long period. Credit: NASA/ESA/G. Bacon.

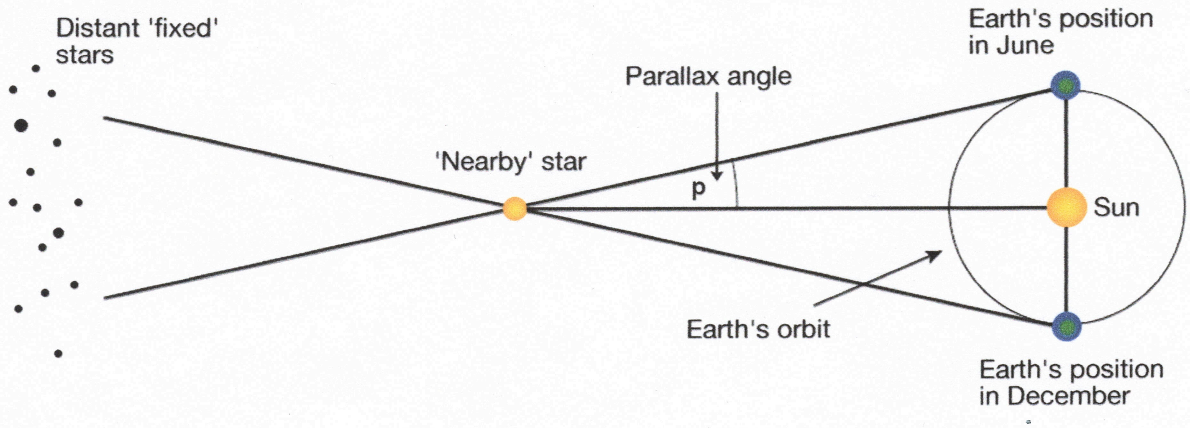

Fig. 13. As the Earth orbits the Sun each year, the position of a nearby star appears to shift with respect to distant stars, which shift imperceptibly. The parallax angle p is larger for nearer stars; a measurement of p (typically much less than 1") directly yields the distance to a star. Diagram <u>not</u> to scale. Credit: summer-astronomy-pc.wikispaces.com/Aristarchus

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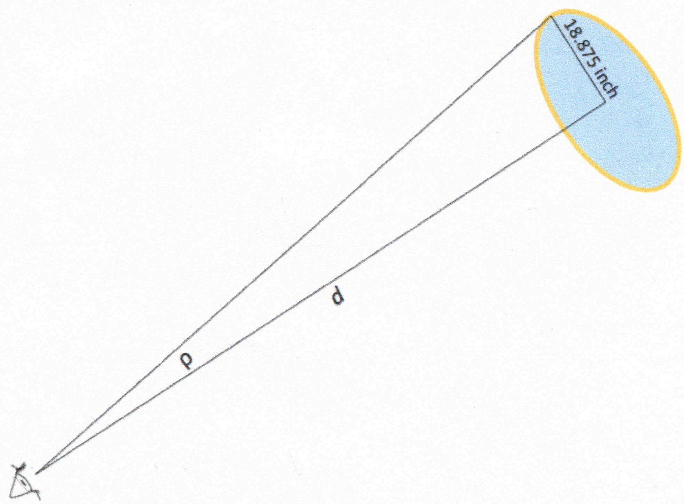

Fig. 14. Geometry for calculating the angular radius of the assumed circular aperture as seen by an observer's eye.

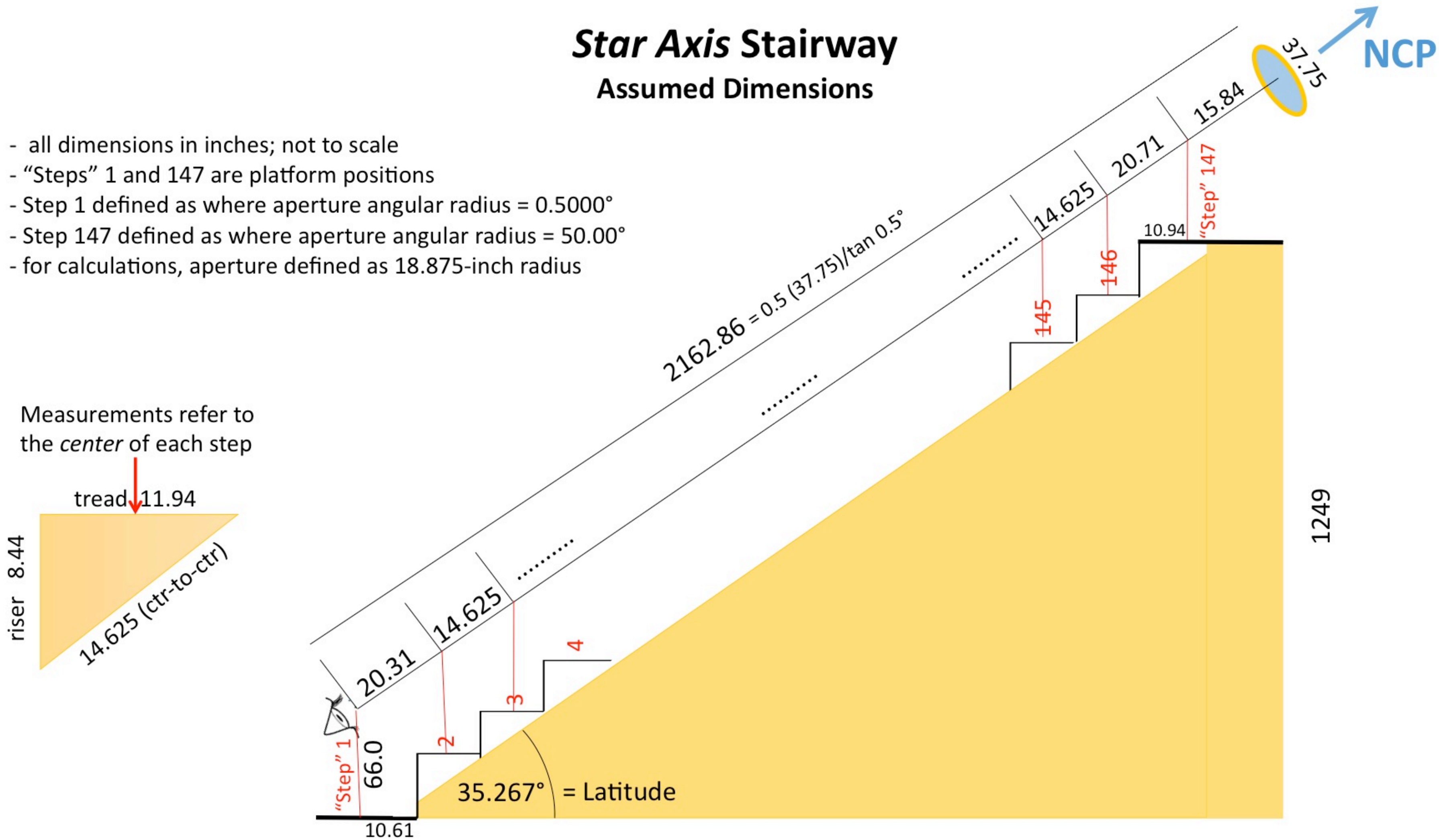

Fig. 15. Dimensions assumed for the stairway of *Star Axis* for the calculation of the stair dates. Many details (such as the Star Chamber at the top) are omitted for clarity. The aperture at the upper right is circular.

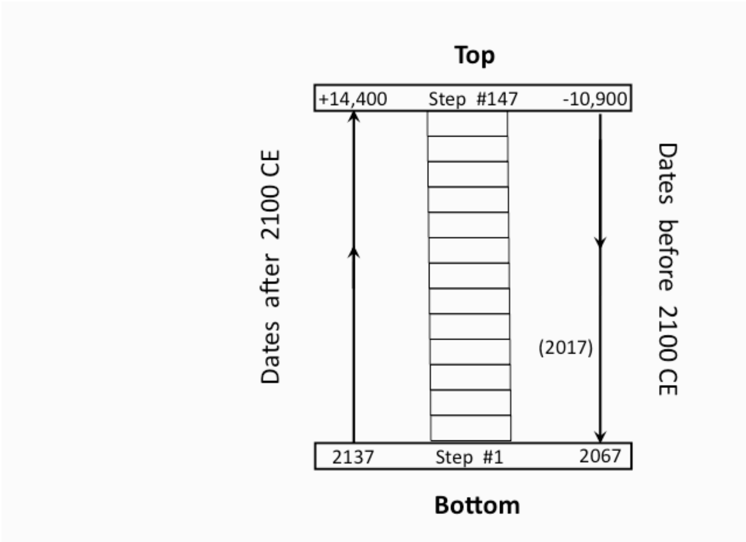

Fig. 16. Schematic view of the run of dates on the *Star Axis* steps. Arrows indicate the direction of time.

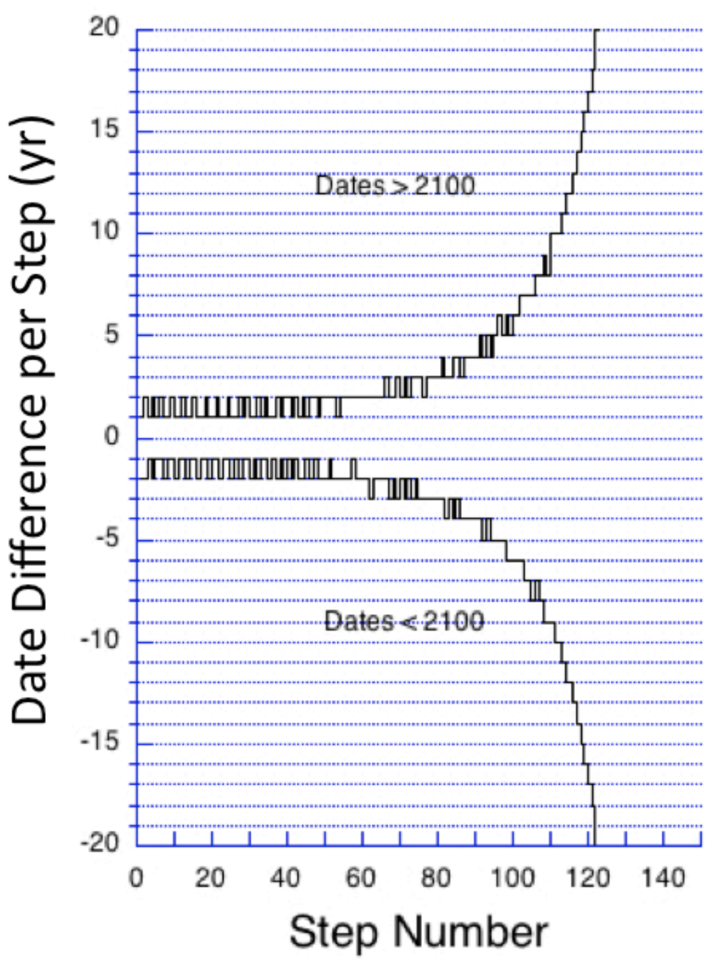

Fig. 17. The difference in years between the dates of adjacent steps. Only Steps #1 through #122 are shown. The "quantization" effects of round-off to the nearest year are clearly seen.